The simplest things: a resistive voltage divider

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We will discuss here the particular case of a resistive DC voltage divider, one of the simplest yet useful circuits you can put together with a couple of resistors (with just one resistor, you can build useful appliances such as a space heater, a clothing iron, or a filament lamp, but these are not very interesting circuits). As you have already seen in class, a voltage divider is a circuit that presents at its output a fraction of the voltage that is applied to its input. The simplest voltage divider is schematically shown in Fig. 1.

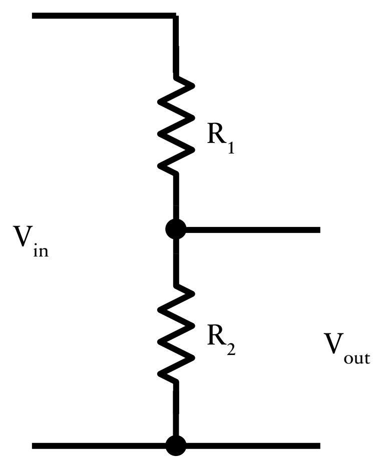


Figure 1. Simplest schematic representation of a voltage divider

So, what is the voltage V_{out}? Well, for the circuit as shown, since the current going through both resistors is $I = V_{in} / (R_1 + R_2)$, the voltage drop across R_2 is

$$V_{out} = IR_2 = V_{in}\left(\frac{R_2}{R_1 + R_2}\right) \tag{1}$$

We will define the attenuation α as the ratio between V_{in} and V_{out} ($\alpha = V_{in} / V_{out}$). The fact that $R_2 \geq 0$ implies $\alpha \geq 1$, meaning that the output voltage is always less than or equal to the input voltage (we will not get distracted here by the intriguing possibility of having components with negative resistance). In particular, we can build a variable voltage divider using a single adjustable resistor split in two – such a resistor is called a potentiometer. One classic application of these adjustable voltage dividers is volume control.

Despite its simplicity, the resistive voltage divider is one of the most common and identifiable sub-circuits you will find in almost any schematic. Figure 2 presents the schematic of a 7-transistor AM radio produced by Hitachi many years ago [1]. It may be too early for a firstyear student to understand how this whole circuit works, but that should not keep you from perusing this and other schematics to see what pieces or functional blocks you can identify. In my mind, reading schematics is not too different from reading sheet music: first, you learn the basic symbols of notation, then the sound of individual notes, then chords, and so on. After some practice, you would be able to read a sheet and hear in your head how the music goes. And after some more practice, you could start writing your own music! With enough practice, a similar thing will happen with electronic components, schematics, and circuit design. (Granted, this is an oversimplification of the process of learning how to read and play sheet music – but I am just an engineer).

Let us go back now to the schematic shown in Fig. 2. It is customary to draw schematics such that signals flow from left to right (at least for somewhat simple schematics). Complex schematics will be typically broken into separate sheets, grouped by purpose or functionality. Schematics can be deceiving: this radio schematic is small enough to fit in a fraction of a page, however understanding all details of how it works will require taking several different courses. In this schematic, the radio signals are captured at the left, and they undergo several transformations until they become audible signals at the right end. This radio is built with resistors, inductors, capacitors, transistors, and diodes – all elements that you will start seeing in this class. For now, I just want you to focus your attention on resistor dividers… how many explicit resistor dividers do you see? Note that R_{14} (in the center of the schematic) is a potentiometer – a single resistor that can be manually adjusted to vary the V_{in}/V_{out} ratio. That potentiometer is attached to the volume knob of this radio. Several other resistor dividers attached to the base terminal (B) of various transistors, are used to *bias* the transistors (you will see more about this in future courses).

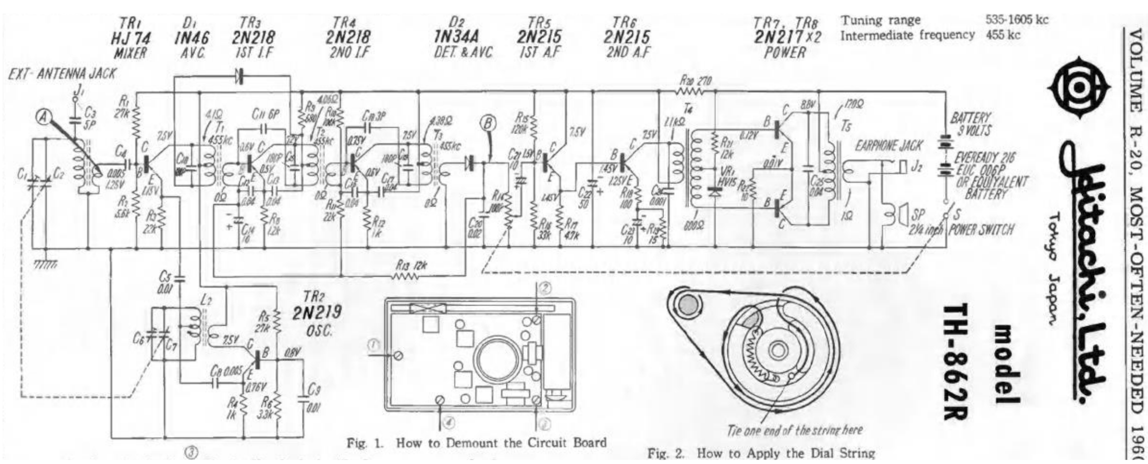


Figure 2. Schematic of a portable, 7-transistor AM radio from Hitachi (1959/1960)

At this point it looks like we know everything there is to be known about voltage dividers: we know their working principle, and we have an equation that relates their input and output voltages. Is that all? Most definitely not. As you may have heard, the devil is in the details… Let say we want to design a resistor divider that will divide (attenuate) the input voltage by a factor of 10 – we want to have a 100V output when 1000V is applied at the input. Isn't it as easy as picking any R_1 and R_2 such that $R_1 = 9R_2$? Well no, not really.

What will be this circuit connected to?

Same as what you observed in the many dividers found in Fig. 2, the resistor divider that you will design will be connected to both input and output networks – other parts of your overall circuit or setup with their own specific characteristics and requirements. Figure 3 is a representation of a more realistic schematic.

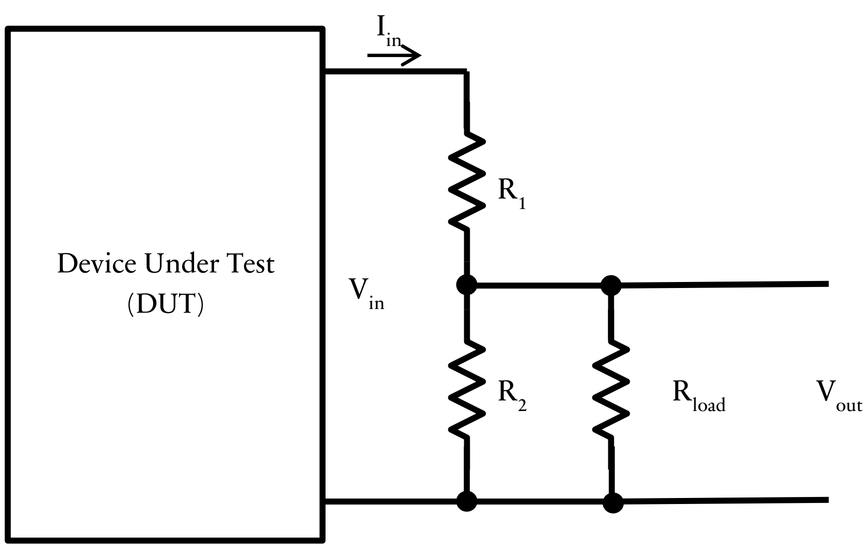


Figure 3. Diagram showing connections of the voltage divider to external circuits

The schematic in Fig. 3 could represent, for example, a case where we want to measure the output voltage of a device under test (for example, this white box could represent the network of components to the left of any of the dividers in Fig. 2) using a voltmeter with a 100 V range. While ideal voltmeters are assumed to have an internal resistance infinitely high (so they would draw no current), real voltmeters have an internal resistance that is typically high, but not infinite. In this case, the current I_{in} can be written as

$$I_{in} = \frac{V_{in}}{R_1 + \frac{R_2 R_{load}}{R_2 + R_{load}}} \qquad (2)$$

and the expression for V_{out} becomes

$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_{load}}}\right) \quad (3)$$

From expression (2) you can see that if $R_{load} >> R_2$ the effect of the load becomes negligible, and we recover expression (1). However, suppose that the effect of the load cannot be neglected. In that case, the actual ratio between input and output voltages will be affected by the extra term in the denominator, and you will infer an input voltage value that is lower than that given by the expression of the ideal divider (1). But how much lower? What is the meaning of negligible? Let us say, for example, that $R_{load} = 10R_1$. Then V_{out} will be

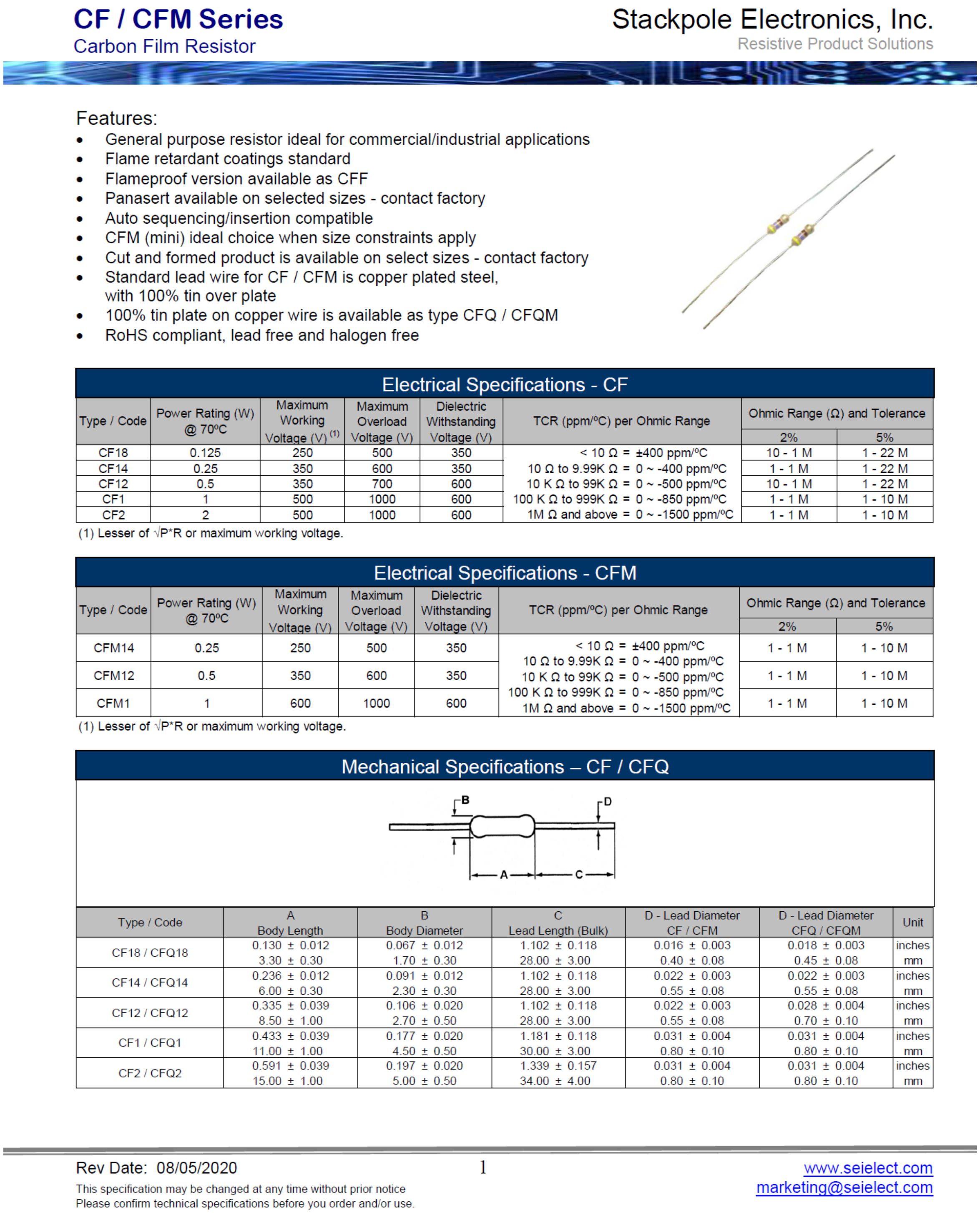
$$V_{out} = V_{in}\left(\frac{R_2}{R_1 + R_2 + 0.1R_2}\right) = V_{in}\left(\frac{R_2}{R_1 + 1.1R_2}\right) \quad (4)$$

Since we already know that $R_1 = 9R_2$, we conclude that $\alpha_{ACTUAL} = 10.1 = 1.01\alpha_{IDEAL}$. So, every time you measure with this same instrument you make a consistent error of -1% in estimating V_{in}. This type of error is called *systematic*, and it can be in principle corrected if you know with enough accuracy the characteristics of your instrument and the resistors in your divider. Is this error negligible? That depends on the specifications of your divider, so besides attenuation, you should have (or be given) a specification on expected accuracy. We will assume here that our accuracy specification is a generous ±10%, so our attenuation specification becomes $\alpha = 10 \pm 1$.

Alright – it looks like the conclusion from the previous paragraph is that the total resistance of the divider should be significantly lower than the internal resistance of the instrument we will use to measure the voltage. Why don't we make R_1 = 900Ω, R_2 = 100Ω? For a typical voltmeter with an internal resistance of 10MΩ, the difference between α_{ACTUAL} and α_{IDEAL} will be in the parts per million. This sounds pretty good, except for the fact that, at a 1000V input, this divider will be drawing 1A from the device under test! Not only that – your voltage divider will be dissipating a power of $P = I^2R = 1000W$! It looks like we need a specification for a maximum current draw; let us assume that the maximum admissible current draw from the device under test is 120μA at 1000V. If we choose a safe design target of 100μA and build a divider with a total resistance of 1000V /100μA = 10MΩ, it will dissipate a maximum power of 100 mW, which is very manageable. Using a 10MΩ voltmeter, we will have a systematic error in our estimation of V_{in} of about -9%, which would be marginally acceptable only if there were no other sources of error (we will see that this is not the case), but in any case, it could be corrected - we will revisit this issue later. How do we calculate this error? If you use R_1 = 9MΩ, R_2 = 1MΩ, and R_{load} = 10MΩ in expression (4), you will conclude that the actual attenuation in this case is $\alpha_{ACTUAL} = 10.9$. So, based on the ideal calculation, for an output voltage of 10V you would predict the input voltage to be 100V, while it is actually 109V. Note here: although it is common to find voltmeters with a 10MΩ internal resistance, other instruments that you may use to measure voltage may have a significantly lower resistance (for example 1MΩ, or even as low as 50Ω, for oscilloscopes).

How are you going to build it?

From the discussion above, it looks like you will need a total resistance of 10MΩ, with a ratio of $R_1 / R_2 = 9$. Will you then choose R_1 = 9MΩ, R_2 = 1MΩ? It looks like a fine choice, except that 9MΩ is not a standard value – you cannot easily buy it, or you will not get it at the price of standard value resistors. Resistors, as every component and material used to build electronic assemblies, are described by what is called a specification sheet, or spec for short. The spec is a list of all parameters that are guaranteed by the manufacturer and are important in the description of the part, including values of its electrical parameters, size, physical construction, and many others. It is essential to understand the specifications of all the parts that you will be using in your designs. Now, you could think that resistors are simple enough that do not require much of a specification at all – but you would be wrong. Figure 4 shows the first of 8 pages of the specification for a particular type of resistor (called carbon film resistors) made by Stackpole Electronics, one of many companies that manufacture these components (you can buy them from distributors such as Digikey or Newark). The specification is one of several documents that vendors provide: additional documents include descriptions of how their parts are tested, articles explaining how to understand different portions of their spec, application notes describing how to use their components as part of other circuits (these are particularly interesting, you should start looking around and getting familiar with them), and others. In the case of resistors from Stackpole, you will find a link to their application notes and datasheets on their main page.



CF / CFM Series
Carbon Film Resistor

Stackpole Electronics, Inc.
Resistive Product Solutions

Features:
- General purpose resistor ideal for commercial/industrial applications
- Flame retardant coatings standard
- Flameproof version available as CFF
- Panasert available on selected sizes - contact factory
- Auto sequencing/insertion compatible
- CFM (mini) ideal choice when size constraints apply
- Cut and formed product is available on select sizes - contact factory
- Standard lead wire for CF / CFM is copper plated steel, with 100% tin over plate
- 100% tin plate on copper wire is available as type CFQ / CFQM
- RoHS compliant, lead free and halogen free

Electrical Specifications - CF

Type / Code	Power Rating (W) @ 70ºC	Maximum Working Voltage (V) (1)	Maximum Overload Voltage (V)	Dielectric Withstanding Voltage (V)	TCR (ppm/ºC) per Ohmic Range	Ohmic Range (Ω) and Tolerance	
						2%	5%
CF18	0.125	250	500	350	< 10 Ω = ±400 ppm/ºC	10 - 1 M	1 - 22 M
CF14	0.25	350	600	350	10 Ω to 9.99K Ω = 0 ~ -400 ppm/ºC	1 - 1 M	1 - 22 M
CF12	0.5	350	700	600	10 K Ω to 99K Ω = 0 ~ -500 ppm/ºC	10 - 1 M	1 - 22 M
CF1	1	500	1000	600	100 K Ω to 999K Ω = 0 ~ -850 ppm/ºC	1 - 1 M	1 - 10 M
CF2	2	500	1000	600	1M Ω and above = 0 ~ -1500 ppm/ºC	1 - 1 M	1 - 10 M

(1) Lesser of √P*R or maximum working voltage.

Electrical Specifications - CFM

Type / Code	Power Rating (W) @ 70ºC	Maximum Working Voltage (V)	Maximum Overload Voltage (V)	Dielectric Withstanding Voltage (V)	TCR (ppm/ºC) per Ohmic Range	Ohmic Range (Ω) and Tolerance	
						2%	5%
CFM14	0.25	250	500	350	< 10 Ω = ±400 ppm/ºC 10 Ω to 9.99K Ω = 0 ~ -400 ppm/ºC	1 - 1 M	1 - 10 M
CFM12	0.5	350	600	350	10 K Ω to 99K Ω = 0 ~ -500 ppm/ºC 100 K Ω to 999K Ω = 0 ~ -850 ppm/ºC	1 - 1 M	1 - 10 M
CFM1	1	600	1000	600	1M Ω and above = 0 ~ -1500 ppm/ºC	1 - 1 M	1 - 10 M

(1) Lesser of √P*R or maximum working voltage.

Mechanical Specifications – CF / CFQ

B D A C

Type / Code	A Body Length	B Body Diameter	C Lead Length (Bulk)	D - Lead Diameter CF / CFM	D - Lead Diameter CFQ / CFQM	Unit
CF18 / CFQ18	0.130 ± 0.012	0.067 ± 0.012	1.102 ± 0.118	0.016 ± 0.003	0.018 ± 0.003	inches
	3.30 ± 0.30	1.70 ± 0.30	28.00 ± 3.00	0.40 ± 0.08	0.45 ± 0.08	mm
CF14 / CFQ14	0.236 ± 0.012	0.091 ± 0.012	1.102 ± 0.118	0.022 ± 0.003	0.022 ± 0.003	inches
	6.00 ± 0.30	2.30 ± 0.30	28.00 ± 3.00	0.55 ± 0.08	0.55 ± 0.08	mm
CF12 / CFQ12	0.335 ± 0.039	0.106 ± 0.020	1.102 ± 0.118	0.022 ± 0.003	0.028 ± 0.004	inches
	8.50 ± 1.00	2.70 ± 0.50	28.00 ± 3.00	0.55 ± 0.08	0.70 ± 0.10	mm
CF1 / CFQ1	0.433 ± 0.039	0.177 ± 0.020	1.181 ± 0.118	0.031 ± 0.004	0.031 ± 0.004	inches
	11.00 ± 1.00	4.50 ± 0.50	30.00 ± 3.00	0.80 ± 0.10	0.80 ± 0.10	mm
CF2 / CFQ2	0.591 ± 0.039	0.197 ± 0.020	1.339 ± 0.157	0.031 ± 0.004	0.031 ± 0.004	inches
	15.00 ± 1.00	5.00 ± 0.50	34.00 ± 4.00	0.80 ± 0.10	0.80 ± 0.10	mm

Rev Date: 08/05/2020 1

This specification may be changed at any time without prior notice
Please confirm technical specifications before you order and/or use.

www.seielect.com
marketing@seielect.com

Figure 4. First page of the specification of carbon film resistors manufactured by Stackpole Electronics

Let us go back to the question of how are you going to build your resistor divider. Besides maximum voltage, maximum current draw, attenuation, and accuracy, there may be several other design constraints or specifications, including cost, size, operating temperature range, etc., that will affect or limit your choices. Assume you will use the CFM type of resistor specified in Figure 4 – what values are available? It depends. One of the most common specifications of a resistor is its tolerance: when you order a resistor, the actual value of the parts you receive may be up to 1%, 2%, or 5% away from the *nominal* value you ordered. The choices for nominal values depend on the tolerance you select; available values fall into what is called an EIA standard decade. The resistors described in Fig. 4 are available in 2% or 5% tolerances, standard values for those ranges are as follows:

10	11	**12**	13	**15**
16	**18**	20	**22**	24
27	30	**33**	36	**39**
43	**47**	51	**56**	62
68	75	**82**	91	**100**

Multiplying or dividing these values by powers of 10 will give you the full range of resistor values, typically from 1Ω all the way to 10MΩ. Although all the values in the table are available for 2% resistors, only the values in bold are available for 5% resistors. Resistors with looser tolerance are somewhat less expensive, but your choices are more limited. It looks like no matter what tolerance you choose, the idea of using just two resistors may not work (although one of the values in the 2% table is 91, very close to what you need). You can, of course, build your divider with a combination of resistors chosen to hit your target values (there are other reasons not to build it with just two, as we will see later). But will either tolerance range satisfy your requirements for accuracy? We need to look deeper into it to get an answer.

Error propagation

How does the tolerance of the parts affect the attenuation of your divider? Let us work out some estimates: remember that we defined the attenuation as $\alpha = (R_1 + R_2) / R_2$. Say we define as $t = \Delta R/R$ the stated tolerance (or maximum relative error) of the part (in this case, $t = 0.05$ or $t = 0.02$). The maximum and minimum attenuation we can expect for the extreme values of R_1, R_2 are as follows:

$$\alpha_{max} = \frac{R_1 + \Delta R_1 + R_2 - \Delta R_2}{R_2 - \Delta R_2} \tag{5}$$

$$\alpha_{min} = \frac{R_1 - \Delta R_1 + R_2 + \Delta R_2}{R_2 + \Delta R_2} \tag{6}$$

From (5) and (6) we can calculate the relative error expected for the attenuation

$$\Delta\alpha/\alpha = \left(\frac{\alpha_{max} - \alpha_{min}}{2}\right)\frac{1}{\alpha} \cong 2t \tag{7}$$

where we assumed that $t << 1$, and that R_1 is significantly larger than R_2 (see the Appendix for a one-step approach to error propagation). This expression tells us that if we build many resistor dividers using 5% resistors, the actual attenuation of the individual dividers could be up to 10% higher or lower than the nominal. This is a *random* error that comes from the fact that the values of the resistors you buy are randomly distributed within the specified tolerance (we will not discuss here the specific shape of this probability distribution). So, using 5% resistors is not a good choice if the design specification for the divider is 10% - we should go with the 2% resistors. A side note here: the values for most parts (resistors, capacitors, inductors, etc.) are specified anywhere from a few percent up to perhaps 10% or 20%, so there is no point in writing down all the figures you get from your calculator – a few significant figures, consistent with your estimated error, will suffice.

Back to the construction

Based on the estimations above we have decided to use 2% resistors. Looking at the standard value table, we note that there should be a 9.1MΩ resistor, so maybe the divider can be put together with just two resistors after all. But let us look at the specifications more closely: first, this manufacturer only makes 2% resistors up to 1MΩ. Second, even if there was a 9.1MΩ CFM14 resistor available, the maximum working voltage across such a resistor needs to be 250V or $\sqrt{P_{max}R}$, whichever lower. If we try and use a 9.1MΩ resistor, the voltage drop across it will be about 900V (for a 1000V input) so such a design would violate the manufacturer's specs. At this point we could consider changing manufacturers, however, there are not many other choices, and none of them seem to allow more than 500V across their parts, so R_1 may need to be split into several resistors anyway.

Thermal considerations

There are many ways to go about solving the issue we found above. One interesting element that may inform our final decision is thermal considerations. If this comes up as a surprise to you do not worry – it is often a surprise for senior students too. The fact is that thermal considerations are critical for the success of any hardware design, either because they affect its reliability, or they otherwise affect its performance to specifications. Look again at the specs in Fig. 4 and you will find a value called TCR, or Temperature Coefficient of Resistance. This value tells you how much the resistance value will change from its value measured at 25°C as the temperature of the part changes. For 1MΩ and above the specification states a worst-case TCR of -1500 parts per million per degree centigrade (ppm/°C). This means that the value of the resistor will drop by up to 15% if the temperature at the resistor increases to 125°C! (by the way, the operating temperature of these resistors is specified as -55°C to +155°C, so 125°C would be acceptable for this part). Thus, it is important to keep the temperature of the parts as low and as uniform as possible. Both ambient temperature and self-heating contribute to the temperature of the resistor – to keep the self-heating the same across all resistors in the divider, it would be best to build it using ten 1MΩ resistors (since the current through all of them is the same, the dissipation on each will be the same). For a maximum current draw of 100μA, each resistor will dissipate 10mW which, for a CFM14 resistor, will result in a temperature rise of just a few degrees (these values can be derived from the full specification). If the ambient temperature around the divider is kept at around 25°C, this temperature rise will not change the value of the resistors by more than about a percent, which is still within our accuracy budget. And the maximum voltage drop on each resistor will be 100V, which is well within specifications.

A final tweak

As you may remember, connecting a 10 MΩ voltmeter would result in a -9% systematic error in the attenuation of the divider. We need to correct this problem so the total of all our error contributions fits within the 10% budget specified. It is not difficult to recalculate the resistor ratios considering the parallel connection shown in Fig. 3. After some quick algebra, and always keeping in mind the specifications for these resistors, an acceptable solution could use nine 910kΩ resistors for R_1, and one 1MΩ resistor for R_2. With a total nominal resistance of 9.1MΩ when connected to the voltmeter, it would draw a maximum current of 110μA. For the worst case, when the total resistance of the divider is 2% below nominal, the current drawn would be 112μA, still within specifications. Since the current is split between R_2 and R_{load} in a 10:1 ratio, the dissipation at R_2 will be only about 10% lower than that of each resistor of R_1 so the difference in their temperature due to self-heating will be much less than a degree. At ambient temperature, the attenuation of the divider when the instrument is connected will be 10 ± 0.4, well within our ±10% specification.

Summary

In conclusion, we have explored the implementation of a simple concept under somewhat realistic specifications. Starting from a simple squiggle in a piece of paper, we have analyzed some of the most important considerations in the design and implementation of a resistive attenuator. You should be able to follow the derivations, fill in the gaps, and explore the links to specifications and application notes - in fact, we encourage you to do so. The voltage divider will be generalized in later courses by using elements other than resistors, and this will lead to many interesting applications such as highpass and lowpass filters. The analysis of voltage dividers used for high-frequency signals or very short pulses will be also very interesting. As noted by Horowitz and Hill [2], the simple voltage divider is also useful as a way of thinking about a circuit. As they explain, the input voltage and upper resistance might represent the output of an amplifier, say, and the lower resistance might represent the input of the following stage. In this case, the voltage divider equation tells you how much signal gets to the input of that last stage.

Closing thoughts

The intent of this document is not to provide a comprehensive design guideline for resistor dividers – there is more (or perhaps less) to consider depending on the specifications and intended application of the divider (you would be surprised to see the construction of one of our 500kV voltage dividers, which is two feet long and is built with a plastic hose filled with a special water solution). It is certainly not our intention either to paint a frightening, overwhelming picture of how complicated may be to design a seemingly simple thing. The intention here is to start showing you how engineering works: some people define engineering in general terms as 'design under constraint'. There is some truth in that statement; engineering is both finding a solution for an existing (or not yet existing!) problem and coming up with an optimal implementation of that solution. Optimal, in this context, can imply meeting several requirements or maximizing/minimizing the value of some design parameter. Independent of what particular subject they choose (circuit design, software, lasers, artificial intelligence, or a million others), or what is their internal motivation, engineers certainly have fun working on both the challenge of coming up with a conceptual solution and the (sometimes harder) challenge of implementing it under the given constraints.

References

[1] M.N. Beitman, "Most-often-needed 1960 radio diagrams" (Supreme publications, 1960)
[2] P. Horowitz and W. Hill, "The art of electronics" (Cambridge University Press, New York, 2019) – Highly recommended as a general reference for electronic design.
[3] J.R. Taylor, "An introduction to error analysis" (University Science Books, California, 1997)

Appendix

There is a one-step process to estimate errors more easily and accurately, which involves partial derivatives (which you have seen, or will see soon) [3]. For a function of two variables, we can write

$$\Delta f(x,y) \le \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y \qquad (A1)$$

In the case of our attenuation α, we can write

$$\Delta\alpha \le \left|\frac{\partial \alpha}{\partial R_1}\right| \Delta R_1 + \left|\frac{\partial \alpha}{\partial R_2}\right| \Delta R_2 = \left(\frac{1}{R_2}\right)\Delta R_1 + \left(\frac{R_1}{R_2^2}\right)\Delta R_2 \qquad (A2)$$

Dividing both sides by α we get

$$\frac{\Delta\alpha}{\alpha} \le \frac{\Delta R_1}{R_1 + R_2} + \frac{R_1}{R_1 + R_2}\frac{\Delta R_2}{R_2} \qquad (A3)$$

This expression reduces to expression (7) when $R_1 >> R_2$, and the relative uncertainty for both resistors is the same.